3.2 Logarithmic Functions

Intro

Solving for an answer	Solving for a base	Solving for an exponent
1. $3^4 = x$	2. $x^2 = 49$	3. $3^x = 729$

Now you try…

4. $2^5 = x$ 5. $x^3 = 125$ 6. $5^x = 3125$

A logarithmic function is the inverse function of an exponential function.

Every logarithmic equation has an equivalent exponential form:

$y = \log_a x$ is equivalent to $x = a^y$

A logarithm is an exponent!

Examples: Write the equivalent exponential equation and solve for *y*.

Logarithmic Equation	**Equivalent Exponential Equation**	**Solution**
$y = \log_2 16$	$16 = 2^y$	$16 = 2^4 \rightarrow y = 4$
$y = \log_2(\frac{1}{2})$	$\frac{1}{2} = 2^y$	$\frac{1}{2} = 2^{-1} \rightarrow y = -1$
$y = \log_4 16$	$16 = 4^y$	$16 = 4^2 \rightarrow y = 2$
$y = \log_5 1$	$1 = 5^y$	$1 = 5^0 \rightarrow y = 0$

3.2 Material

- Logarithms are used when solving for an exponent.

$$\text{base}^{\text{exp}} = \text{ans} \leftrightarrow \log_{\text{base}} \text{ans} = \text{exp}$$

- In your calculator: log is base 10 ln is base *e*

Properties of Logarithms and Ln

1. $\log_a 1 = 0$ $\rightarrow a^0 = 1$
2. $\log_a a = 1$ $\rightarrow a^1 = a^1$
3. $\log_a a^x = x$ $\rightarrow a^x = a^x$

- The graph is the inverse of $y=a^x$ (reflected over $y=x$)

VA: $x = 0$, x-intercept (1,0), increasing

Examples: Calculate the values using a calculator.

Function Value	Keystrokes	Display
$\log_{10} 100$	LOG 100 ENTER	2
$\log_{10}(\frac{2}{5})$	LOG (2 ÷ 5) ENTER	– 0.3979400
$\log_{10} 5$	LOG 5 ENTER	0.6989700
$\log_{10} -4$	LOG –4 ENTER	ERROR

no power of 10 gives a negative number

Properties of Logarithms

1. $\log_a 1 = 0$ since $a^0 = 1$.
2. $\log_a a = 1$ since $a^1 = a$.
3. $\log_a a^x = x$
4. If $\log_a x = \log_a y$, then $x = y$. one-to-one property

Examples:

1. Solve for x: $\log_6 6 = x$ → $6^x = 6$, $x = 1$

2. Simplify: $\log_3 3^5$ → $x = 5$ → $3^x = 3^5$

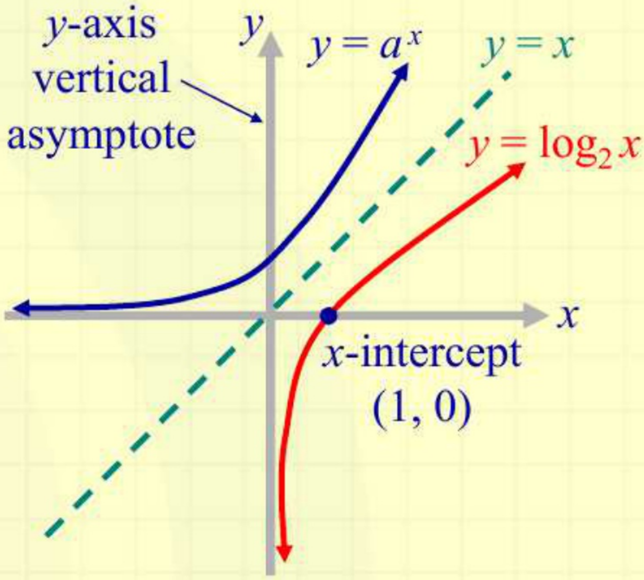
The graphs of logarithmic functions are similar for different values of a.

$$f(x) = \log_a x$$

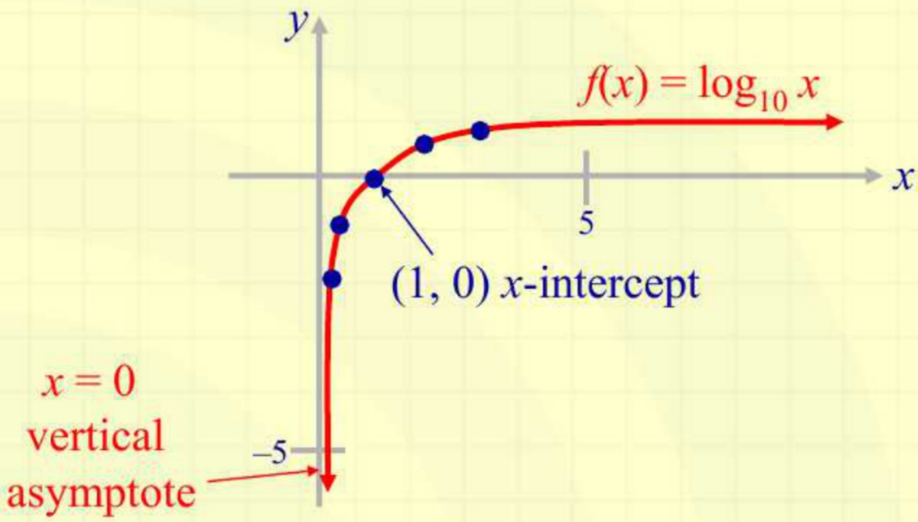
Graph of $f(x) = \log_a x$

x-intercept (1, 0)
VA: x = 0
increasing

reflection of $y = a^x$ in $y = x$

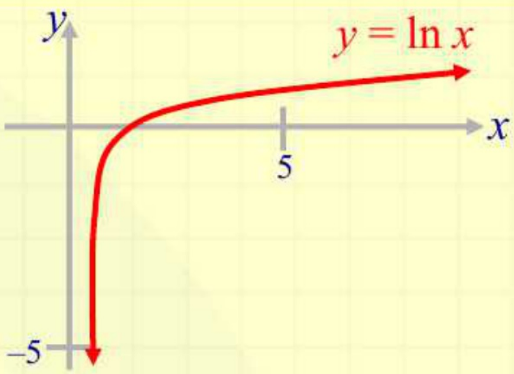


Example: Graph the common logarithm function $f(x) = \log_{10} x$.



The function defined by
$f(x) = \log_e x = \ln x$

is called the **natural logarithm function**.



$y = \ln x$ is equivalent to $e^y = x$

Use a calculator to evaluate: ln 3, ln –2, ln 100

Function Value	Keystrokes	Display
ln 3	LN 3 ENTER	1.0986122
ln –2	LN –2 ENTER	ERROR
ln 100	LN 100 ENTER	4.6051701

Properties of Natural Logarithms

1. $\ln 1 = 0$ since $e^0 = 1$.
2. $\ln e = 1$ since $e^1 = e$.
3. $\ln e^x = x$
4. If $\ln x = \ln y$, then $x = y$. one-to-one property

$\log_e(x) = \ln$

Examples: Simplify each expression.

$\ln\left(\frac{1}{e^2}\right) = \ln\left(e^{-2}\right) = -2$ inverse property

$3 \ln e = 3(1) = 3$ property 2

$\sqrt{\ln 1} = \sqrt{0} = 0$ property 1

Classwork

- Pg 236 # ~~1-21 odd~~
  39-44, 65-68, 79-85 odd

Example: The formula $R = \left(\frac{1}{10^{12}}\right)e^{\frac{-t}{8223}}$ (t in years) is used to estimate the age of organic material. The ratio of carbon 14 to carbon 12 in a piece of charcoal found at an archaeological dig is $R = \frac{1}{10^{15}}$.

How old is it?

$$\left(\frac{1}{10^{12}}\right)e^{\frac{-t}{8223}} = \frac{1}{10^{15}}$$ original equation

$$e^{\frac{-t}{8223}} = \frac{1}{10^{3}}$$ multiply both sides by 10^{12}

$$\ln e^{\frac{-t}{8223}} = \ln \frac{1}{1000}$$ take the ln of both sides

$$\frac{-t}{8223} = \ln \frac{1}{1000}$$ inverse property

$$t = -8223\left(\ln \frac{1}{1000}\right) \approx -8223(-6.907) = 56796$$

To the nearest thousand years the charcoal is 57,000 years old.

3.2 Material

- Logarithms are used when solving for an exponent.

$$\log_a b = c \leftrightarrow a^c = b$$

- In your calculator: log is base 10 ln is base *e*

Properties of Logarithms and Ln

1. $\log_a 1 = 0$
2. $\log_a a = 1$
3. $\log_a a^x = x$

- The graph is the inverse of $y=a^x$ (reflected over $y=x$)

VA: $x = 0$, x-intercept (1,0), increasing

Practice Problems

- Page 236 #27–29, 32-38, 80-86 even

$$3^{x+5} = 8^{x-3}$$

$$\log_3 3^{x+5} = \log_3 8^{x-3}$$

$$x+5 = \log_3 8^{x-3}$$

$$x+5 = (x-3)\cdot\log_3 8$$

$$x+5 = x\cdot\log_3 8 - 3\cdot\log_3 8$$

$$5+3\cdot\log_3 8 = x\cdot\log_3 8 - x$$

$$5+3\cdot\log_3 8 = x(\log_3 8 - 1)$$

$$x = \frac{5+3\cdot\log_3 8}{\log_3 8 - 1}$$

pg 236 #27-29
32-38
80-86 ev

1 1 2 3 5 8 13